Application of Differential Transform Method Stiff Systems

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ABSTRACT

In this paper, Differential Transform Method (DTM) is proposed for the closed form solution of linear and non-linear stiff systems. This method gives the series of solutions which can easily be converted into exact solution. The method is described and illustrated with the help of examples. The results show that DTM is very easy, effective and convenient.

Key Words: Differential Transform, Method, Stiff System, Inverse Differential Transform and Applicaiton to Stiff System

INTRODUCTION

Consider the stiff initial value problem (Hojjati, Rahimi Arabili & Hosseini ,1996).

$$y'(x) = f(x,(x)), \quad y(x_0) = y_0 \tag{1}$$

on the finite interval $I = [x_0, x_N]$, where $y : [x_0, x_N] \to R^m$ and $f : [x_0, x_N] \times R^m \to R^m$ are continuous.

"When the solution of the system contains components which change at significantly different rates for given changes in the independent variable, the system is said to be stiff" (Harier & G.Wanner, 1976), (Lopidus & Schiesser ,1976), (Hojjati, Rahimi Arabili & Hosseini ,1996), (Harier & Wanner, 1976), (Lopidus, Schiesser,1976), (Carroll, 1993) and (Hsiao & Haar, 2004), particularly, in the fields of electrical circuits, vibrations and chemical reactions. Stiff differential equations are characterized as those whose exact solution has a term of the form.

ONE-DIMESNIONAL DIFFERENTIAL TRANSFORM

Differential transform of a function

$$Y(k) = \frac{1}{k!} \frac{d^k y}{dx^k}\bigg|_{x=0} \tag{2}$$

where $k = 0, 1, 2, 3, \ldots$

where $y(x)$ is the original function and $Y(k)$ is the transformed function. The Differential inverse transform of $Y(k)$ is defined as

$$y(x) = \sum_{k=0}^{\infty} x^k Y(k) \tag{3}$$

From equation (2) and (3) we get

$$y(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!} \frac{d^k y}{dx^k}\bigg|_{x=0} \tag{4}$$

which implies that the concept of DTM is derived from Taylor series expansion, but the method does not evaluate the derivative symbolically? However, relative derivatives are calculated by an iterative procedure which is described by the transformed equations of the original functions. In this paper, we use the lower case letters to represent the original functions and an upper case letter to represent the transformed functions. In actual applications, the function y (x) is expressed by a finite series equation (3) can be written as

$$y(x)=\sum_{k=0}^{n} x^k Y(k) \tag{5}$$

Here n is decided by the convergence of natural frequency in this study.
From definition (2) and (3), it can easily obtained the Table 1 of the fundamental operations of one-dimensional differential transform method as:

Table1.
The fundamental operations of one-dimensional differential transform method

Original function	Transformed function
$y(x)=u(x)\pm v(x)$	$Y(k)=U(k)\pm V(k)$
$y(x)=cw(x)$	$Y(k)=cW(k)$
$y(x)={dy}/{dx}$	$Y(k)=(k+1)W(k+1)$
$y(x)={d^j y}/{dx^j}$	$Y(k)=(k+1)(k+2)...(k+j)W(k+j)$
$y(x)=u(x)v(x)$	$Y(k)=\sum_{r=0}^{k} U(r)V(k-r)$
$y(x)=\exp(\lambda x)$	$Y(k)=\dfrac{\lambda^k}{k!}$

APPLICTION TO STIFF SYSTEM

In this section we will apply DTM to linear and non-linear stiff systems:

Problem 1: Consider the linear stiff system:

$$y_1'=-y_1-15y_2+15e^{-x}, \tag{6}$$

$$y_2'=15y_1-y_2-15e^{-x} \tag{7}$$

With initial value $y_1(0)=1$, $y_2(0)=1$

This system has eigenvalues of large modulus lying closed to the imaginary axis $-1\pm 15I$.
Applying Differential Transformation we have

$$Y_1(k+1)=\frac{1}{k+1}\left[-Y_1(k)-15Y_2(k)+15\frac{(-1)^k}{k!}\right] \tag{8}$$

$$Y_2(k+1)=\frac{1}{k+1}\left[15Y_1(k)-Y_2(k)-15\frac{(-1)^k}{k!}\right] \tag{9}$$

Differential Transformation of the initial conditions (3)

$$Y_1(0)=1, Y_2(0)=1$$

For $k=0,1,2,3,...$ the series coefficients for $Y_1(k)$ and $Y_2(k)$ can be obtained as

$$Y_1(0)=1\ ,\ Y_1(1)=-1\ , Y_1(2)=\frac{1}{2!}\ ,\ Y_1(3)=-\frac{1}{3!}, Y_1(4)=\frac{1}{4!},\ Y_1(5)=-\frac{1}{5!},....$$

$$Y_2(0)=1,\ \ Y_2(1)=-1,\ Y_2(2)=\frac{1}{2!}\ ,\ Y_2(3)=-\frac{1}{3!}, Y_2(4)=\frac{1}{4!},\ Y_2(5)=-\frac{1}{5!},....$$

We have used the MATHEMATICA to calculate the unknown coefficients $Y_1(k)$ and $Y_2(k)$.
Using the inverse Transform:

$$y(x)=\sum_{k=0}^{\infty} x^k Y(k) \tag{10}$$

$$y_1(x)=1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-\frac{x^5}{5!}+... \tag{11}$$

$$y_2(x)=1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-\frac{x^5}{5!}+... \tag{12}$$

i-e $$y_1(x)=e^{-x} \tag{13}$$

$$y_2(x)=e^{-x} \tag{14}$$

Thus we get the exact solution by differential transform method.

Problem 2: Consider the non-linear initial value problem [1]

$$y_1'=-1002y_1+1000y^2{}_2\ ,\quad y_1(0)=1 \tag{15}$$

$$y_2'=y_1-y_2-y^2{}_2\ ,\qquad y_2(0)=1 \tag{16}$$

Applying Differential Transform, we have

$$Y_1(k+1)=\frac{1}{(k+1)}\left[-1002Y_1(k)+1000\sum_{r=0}^{k} Y_1(r)Y_1(k-r)\right] \tag{17}$$

$$Y_2(k+1)=\frac{1}{(k+1)}[Y_1(k)-Y_2(k)-\sum_{r=0}^{k} Y_1(r)Y_1(k-r)] \tag{18}$$

For $k=0,1,2,3,...,n$ the series coefficients for $Y_1(k)$ and $Y_2(k)$ can be obtained as

$$Y_1(0)=1\ ,\ Y_1(1)=-2\ , Y_1(2)=2\ ,\ Y_1(3)=-\frac{4}{3}, Y_1(4)=-\frac{2}{3},\ ....$$

$$Y_2(0)=1,\ \ Y_2(1)=-1,\ Y_2(2)=\frac{1}{2}\ ,\ Y_2(3)=-\frac{1}{3!}, Y_2(4)=\frac{1}{4!},\ Y_2(5)=-\frac{1}{5!},....$$

We have used the MATHEMATICA to calculate the unknown coefficients $Y_1(k)$ and $Y_2(k)$.
Using the inverse Transform:

$$y(x)=\sum_{k=0}^{\infty} x^k Y(k) \tag{19}$$

$$y_1(x)=1-\frac{2x}{1!}+\frac{4x^2}{2!}-\frac{8x^3}{3!}+\frac{16x^4}{4!}-... \tag{20}$$

$$y_2(x)=1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-\frac{x^5}{5!}+... \tag{21}$$

i-e $$y_1(x)=e^{-2x} \tag{22}$$

$$y_2(x)=e^{-x} \tag{23}$$

Thus we get the exact solution by differential transform method.

CONCLUSION

In this research article, differential transform method was applied to the exact solution of linear and non-linear stiff system. Actually DTM is a numerical method and we obtained a closed form solution by a numerical method. It is clear from the references (Fatma Ayaz, 2003) that sometimes it works very well and we obtained a closed form solution. We observed that all the calculations were easy and reliable.

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